Formulario de Electromagnetismo[1]

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Profesor: Gonzalo Palma - Auxiliares: Diego García - Esteban Rodríguez

Departamento de Física, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile

1. Coordenadas Cartesianas

1.1. Cantidades cinemáticas

Posición:	$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
Velocidad:	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$
Aceleración:	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

1.2. Elementos diferenciales

Línea:	$d\vec{l} = dx\hat{\jmath} + dy\hat{\jmath} + dz\hat{k}$
Superficie:	$d\vec{S} = dydz\hat{\imath} + dxdz\hat{\jmath} + dydx\hat{k}$
Volumen:	$dV = dxdydz$

1.3. Operadores vectoriales

Gradiente:	$\nabla\varphi = \frac{\partial\varphi}{\partial x}\hat{\imath} + \frac{\partial\varphi}{\partial y}\hat{\jmath} + \frac{\partial\varphi}{\partial z}\hat{k}$
Rotor:	$\nabla\times\vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$
Divergencia:	$\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
Laplaciano:	$\nabla^2\varphi = \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}$

2. Coordenadas Cilíndricas

2.1. Cantidades cinemáticas

Posición:	$\vec{r} = \rho\hat{\rho} + z\hat{k}$
Velocidad:	$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{k}$
Aceleración:	$\vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right)\hat{\phi} + \ddot{z}\hat{k}$
Aceleración:	$\vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\rho} + \frac{1}{\rho}\frac{d}{dt}(\rho^2\dot{\phi})\hat{\phi} + \ddot{z}\hat{k}$

2.2. Equivalencias con coordenadas cartesianas

$\hat{\rho} = \cos\phi\hat{\imath} + \operatorname{sen}\phi\hat{\jmath}$	$\hat{\imath} = \cos\phi\hat{\rho} - \operatorname{sen}\phi\hat{\phi}$
$\hat{\phi} = -\operatorname{sen}\phi\hat{\imath} + \cos\phi\hat{\jmath}$	$\hat{\jmath} = \operatorname{sen}\phi\hat{\rho} + \cos\phi\hat{\phi}$

[1] Si encuentran algún error o ecuación que valga la pena poner, les pido por favor que me lo informen.

2.3. Elementos diferenciales

Línea:	$d\vec{l} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{k}$
Superficie:	$d\vec{S} = \rho d\phi dz\hat{\rho} + d\rho dz\hat{\phi} + \rho d\rho d\phi\hat{k}$
Volumen:	$dV = \rho d\rho d\phi dz$

$dA = \rho \cdot d\rho \cdot d\phi$

2.4. Operadores vectoriales

Gradiente:	$\nabla\varphi = \frac{\partial\varphi}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial\varphi}{\partial\phi}\hat{\phi} + \frac{\partial\varphi}{\partial z}\hat{k}$
Rotor:	$\nabla\times\vec{F} = \left[\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{\rho} + \left[\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right]\hat{\phi} + \frac{1}{\rho}\left[\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{\partial F_\rho}{\partial\phi}\right]\hat{k}$
Divergencia:	$\nabla\cdot\vec{F} = \frac{1}{\rho}\frac{\partial(\rho F_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$
Laplaciano:	$\nabla^2\varphi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\varphi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\varphi}{\partial\phi^2} + \frac{\partial^2\varphi}{\partial z^2}$

3. Coordenadas Esféricas

3.1. Cantidades cinemáticas

Posición:	$\vec{r} = r\hat{r}$
Velocidad:	$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\,\mathrm{sen}\,\theta\hat{\phi}$
Aceleración:	$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\,\mathrm{sen}^2\,\theta\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\,\mathrm{sen}\,\theta\cos\theta\right)\hat{\theta} + \left(r\ddot{\phi}\,\mathrm{sen}\,\theta + 2\dot{r}\dot{\phi}\,\mathrm{sen}\,\theta + 2r\dot{\phi}\dot{\theta}\cos\theta\right)\hat{\phi}$
Aceleración:	$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\,\mathrm{sen}^2\,\theta\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\,\mathrm{sen}\,\theta\cos\theta\right)\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{d}{dt}\left(r^2\dot{\phi}\,\mathrm{sen}^2\theta\right)$

3.2. Equivalencias con cartesianas

$\hat{r} = (\cos\phi\hat{\imath} + \mathrm{sen}\,\phi\hat{\jmath})\,\mathrm{sen}\,\theta + \hat{k}\cos\theta$	$\hat{\imath} = \mathrm{sen}\,\theta\cos\phi\hat{r} + \cos\theta\cos\phi\hat{\theta} - \mathrm{sen}\,\phi\hat{\phi}$
$\hat{\theta} = (\cos\phi\hat{\imath} + \mathrm{sen}\,\phi\hat{\jmath})\cos\theta - \hat{k}\,\mathrm{sen}\,\theta$	$\hat{\jmath} = \mathrm{sen}\,\theta\,\mathrm{sen}\,\phi\hat{r} + \cos\theta\,\mathrm{sen}\,\phi\hat{\theta} + \cos\phi\hat{\phi}$
$\hat{\phi} = -\hat{\imath}\,\mathrm{sen}\,\phi + \hat{\jmath}\cos\phi$	$\hat{k} = \hat{r}\cos\theta - \hat{\theta}\,\mathrm{sen}\,\theta$

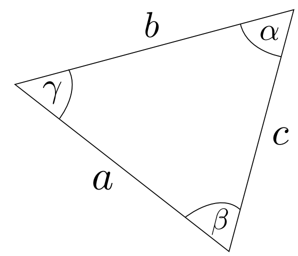
3.3. Elementos diferenciales

Línea:	$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\,\mathrm{sen}\,\theta\hat{\phi}$
Superficie:	$d\vec{S} = r^2\,\mathrm{sen}\,\theta d\theta d\phi\hat{r} + r\,\mathrm{sen}\,\theta dr d\phi\hat{\theta} + rd\theta dr\hat{\phi}$
Volumen:	$dV = r^2\,\mathrm{sen}\,\theta dr d\phi d\theta$

3.4. Operadores vectoriales

Gradiente:	$\nabla\varphi = \frac{\partial\varphi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\varphi}{\partial\phi}\hat{\phi}$
Rotor:	$\nabla\times\vec{F} = \frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial(\mathrm{sen}\,\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\mathrm{sen}\,\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(rF_\phi)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right]\hat{\phi}$
Divergencia:	$\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta F_\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial F_\phi}{\partial\phi}$
Laplaciano:	$\nabla^2\varphi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) + \frac{1}{r^2\,\mathrm{sen}\,\theta}\frac{\partial}{\partial\theta}\left(\mathrm{sen}\,\theta\frac{\partial\varphi}{\partial\theta}\right) + \frac{1}{r^2\,\mathrm{sen}^2\,\theta}\frac{\partial^2\varphi}{\partial\phi^2}$

4. Geometría



Teo. seno:	$\frac{\mathrm{sen}\,\alpha}{a} = \frac{\mathrm{sen}\,\beta}{b} = \frac{\mathrm{sen}\,\gamma}{c}$
Teo. coseno:	$c^2 = a^2 + b^2 - 2ab\cos\gamma.$

5. Cálculo Vectorial

5.1. Teoremas de cálculo vectorial

Teo. Gauss:	$\iint_D \vec{F}\cdot d\vec{S} = \iiint_U \left(\nabla\cdot\vec{F}\right) dV$
Teo. Stokes:	$\iint_D \left(\nabla\times\vec{F}\right)\cdot d\vec{S} = \int_{\partial D}\vec{F}\cdot d\vec{l}$

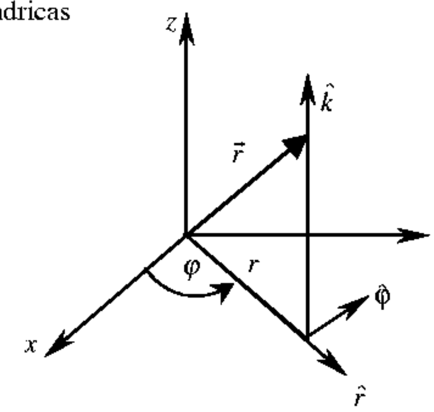
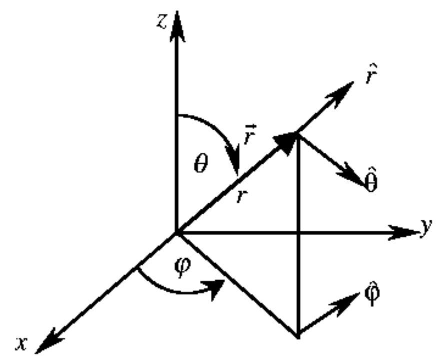
5.2. Identidades vectoriales útiles

- $\nabla\times(\nabla\varphi) = 0.$
- $\nabla\cdot\left(\nabla\times\vec{F}\right) = 0.$
- $\nabla\times\left(\nabla\times\vec{F}\right) = \nabla\left(\nabla\cdot\vec{F}\right) - \nabla^2\vec{F}.$
- $\nabla\times(f(r)\hat{r}) = 0.$
- $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}.$
- $\nabla^2\left(\frac{1}{r}\right) = \delta(\vec{r}),\ r\neq 0.$
- $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}\,(\vec{a}\cdot\vec{c}) - \vec{c}\left(\vec{a}\cdot\vec{b}\right).$

6. Ecuaciones de Maxwell

Ley de Gauss:	$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$
	$\nabla\cdot\vec{B} = 0$
Ley de Faraday:	$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$
Ley de Ampère generalizada:	$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

Formulario Matemático de Electromagnetismo

C.Cilíndricas	C. Esféricas
	

Cilíndricas:

$$\vec{r} = r\hat{r} + z\hat{k}$$
$$x = r\cos\varphi$$
$$y = r\,\mathrm{sen}\,\varphi$$
$$z = z$$

Esféricas:

$$\vec{r} = r\hat{r}$$
$$x = r\,\mathrm{sen}\,\theta\cos\varphi$$
$$y = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi$$
$$z = r\cos\theta$$

1. Gradientes

Cartesianas

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

Cilíndricas

$$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\varphi}\hat{\varphi} + \frac{\partial\phi}{\partial z}\hat{k}$$

Esféricas

$$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\phi}{\partial\varphi}\hat{\varphi}$$

2. Divergencias

Cartesianas

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cilíndricas

$$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

Esféricas

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

3. Rotores

Cartesianas

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

Cilíndricas

$$\nabla\times\vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\varphi} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ A_r & rA_\varphi & A_z \end{vmatrix} = \frac{1}{r}\left\{\left(\frac{\partial A_z}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial z}\right)\hat{r} + r\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\varphi} + \left(\frac{\partial(rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right)\hat{k}\right\}$$

Esféricas

$$\nabla\times\vec{A} = \frac{1}{r^2\,\mathrm{sen}\,\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\,\mathrm{sen}\,\theta\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_r & rA_\theta & r\,\mathrm{sen}\,\theta A_\varphi \end{vmatrix}$$

$$= \frac{1}{r^2\,\mathrm{sen}\,\theta}\left\{\left(\frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial\theta} - \frac{\partial(rA_\theta)}{\partial\varphi}\right)\hat{r} + \left(\frac{\partial A_r}{\partial\varphi} - \frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial r}\right)\hat{\theta} + \left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)r\,\mathrm{sen}\,\theta\hat{\varphi}\right\}$$

4. Laplacianos

Cartesianas	Cilíndricas
$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$	$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$
Esféricas	
$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$	

5. Elementos diferenciales

De línea		
Cartesianas	Cilíndricas	Esféricas
$d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$	$d\vec{l} = dr\hat{r} + rd\varphi\hat{\varphi} + dz\hat{k}$	$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\operatorname{sen}\theta d\varphi\hat{\varphi}$
De superficie		
Cartesianas	Cilíndricas	Esféricas
$d\vec{s} = dydz\hat{i} + dxdz\hat{j} + dxdy\hat{k}$	$d\vec{s} = rd\varphi dz\hat{r} + drdz\hat{\varphi} + rdrd\varphi\hat{k}$	$d\vec{s} = r^2\operatorname{sen}\theta d\theta d\varphi\hat{r} + r\operatorname{sen}\theta drd\varphi\hat{\theta} + rd\theta dr\,\hat{\varphi}$
De volumen		
Cartesianas	Cilíndricas	Esféricas
$dv = dxdydz$	$dv = rdrd\varphi dz$	$dv = r^2\operatorname{sen}\theta drd\varphi d\theta$

donde:

en cartesianas $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

en cilíndricas $\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_z\hat{k}$

en esféricas $\vec{A} = A_r\hat{r} + A_\varphi\hat{\varphi} + A_\theta\hat{\theta}$

6. Identidades Vectoriales

$\nabla\times(\nabla\phi) = 0$

$\nabla\cdot(\nabla\times\vec{A}) = 0$

$\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$

$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(\phi\vec{A}) = \phi\nabla\cdot\vec{A} + \vec{A}\cdot\nabla\phi$

$\nabla\times(f(r)r) = 0$

$\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$ $\quad(\text{con}\,|\vec{r}| = r)$

$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B})$

$\nabla\cdot\vec{r} = 3 \qquad \nabla\times\vec{r} = 0$

$\nabla(\vec{A}\cdot\vec{r}) = \vec{A}$

$\nabla\times(\phi\vec{A}) = \nabla\phi\times\vec{A} + \phi(\nabla\times\vec{A})$

$\nabla(r^n) = nr^{n-2}\vec{r}$

$\nabla^2\left(\frac{1}{r}\right) = \delta(\vec{r})$

$\nabla^2\left(\frac{1}{r}\right) = 0 \qquad (\text{para} \quad r \neq 0)$

$\nabla\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\nabla)\vec{A} - (\vec{A}\cdot\nabla)\vec{B} + \vec{A}\nabla\cdot\vec{B} - B\nabla\cdot\vec{A}$

$\nabla\times(\phi\vec{A}) = \phi\nabla\times\vec{A} - \vec{A}\times\nabla\phi$