Primordial curvature perturbation and spectator fields statistics

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1 Introduction

Hot Big Bang cosmology considers that the universe started expanding from a hot and dense initial state. This theory has been supported by two principal observations: the relation between distance and redshift of galaxies, and by temperature fluctuations of the cosmic microwave background (CMB) [1].

A way to obtain the initial state that the Hot Big Bang requires, is to consider a period of rapid expansion of the universe at early times, which we call it "inflation" [2]. During this inflation era quantum fluctuations evolved, and, in the case of the primordial curvature perturbation, they seed the gravitational collapse distribution that produces the fluctuations of the CMB and gave birth to the first stars which later formed the galaxies.

A simple inflation model is the slow-roll single-field inflation model. This model predicts a perfect Gaussian probability density function (PDF) for the curvature perturbation. However, in order to explore different inflation mechanisms, in this thesis we will consider a multi-field inflation model that is described by the usual Einstein-Hilbert contribution [3] and a derivative coupling between primordial perturbation and a spectator field [4].

To compute the PDF of the primordial perturbation, or the PDF of other fields interacting with it, principally, first, we will compute the correlation function associated [5]. Usually, these functions are divergent due to the perturbatively method that we use, thus is necessary to regularize the theory. We will do this using two methods: Schwinger-Keldysh path integral formalism [6], and wave function of the Universe formalism [7].

The computation of n-point correlation functions, with n an arbitrary natural number, allows us to get small corrections to the leading Gaussian distribution. Nevertheless, an important correlation function to contrast with observations of the CMB temperature fluctuations, is the 2-point correlation function, commonly called power spectrum [8]. In this work we will compute the power spectrum for a 2-fields model with a constant coupling between the fields.

2 Multi-field inflation model

We start from a general action for a multifield inflation model, where we must consider a contribution from space-time curvature S_{EH} and interactions between scalar fields S_ϕ [4],

$$\begin{aligned} S &= S_{\mathrm{EH}} + S_\phi \\ &= \int \mathrm{d}^4x \sqrt{-g}R + \int \mathrm{d}^4x \sqrt{-g}\left[-\frac{1}{2}\gamma_{ab}(\phi)\partial_\mu\phi^a\partial^\mu\phi^b - V(\phi)\right], \end{aligned} \tag{1}$$

where $g_{\mu\nu}$ is the space-time metric, g its determinant, R the Ricci scalar, γ_{ab} the metric that defines the geometry of the field-space, and $V(\phi)$ the potential with interactions between fields without derivative terms. We will be working with a 2-fields model and using perturbation theory.

Approximating up to the second order [9], we obtain the Lagrangian

$$\mathcal{L}(\zeta,\psi) = a^4\epsilon\left(\dot{\zeta} - \alpha\psi\right)^2 - a^2\epsilon(\nabla\zeta)^2 + \frac{1}{2}a^4\dot{\psi}^2 - \frac{1}{2}a^2(\nabla\psi)^2 - a^4\Delta V(\psi), \tag{2}$$

where ζ is the primordial curvature perturbation, that is introduced as a perturbation of the metric; ψ is the isocurvature field; $\Delta V(\psi)$ is the potential representing the self-interaction of ψ; ϵ the slow-roll parameter; and $a(t)$ the scale factor. In this thesis work we will consider it as an arbitrary function expanded in Taylor series,

$$\Delta V(\psi) = \sum_{m=0}^{\infty}\frac{\lambda_m}{m!}\psi^m, \tag{3}$$

fulfilling the condition

$$\Delta V(\psi) \ll H^4, \tag{4}$$

with H the Hubble parameter.

3 Methodology

As we mentioned in the introduction section, we have three ways to analyze the statistics of the primordial curvature perturbation: Schwinger-Keldysh in-in path integral formalism, wavefunction of the Universe, and Bogoliubov coefficients. Now we will mention a brief introduction and the current state of each method.

3.1 n-point correlation function

We will use Schwinger-Keldysh (S-K) path integral representation to compute the correlation function using diagrammatic representation [6]. As in a common path integral formalism, we calculate the expectation value of an operator (in our case: $n-$point functions) taking functional derivatives from the generating functional $Z[J_\pm]$,

$$\langle \varphi(\tau, \mathbf{x}_1) \cdots \varphi(\tau, \mathbf{x}_n) \rangle = \frac{\delta}{i\delta J_+(\tau, \mathbf{x}_1)} \cdots \frac{\delta}{i\delta J_+(\tau, \mathbf{x}_n)} Z[J_\pm]\Big|_{J_\pm = 0} . \tag{5}$$

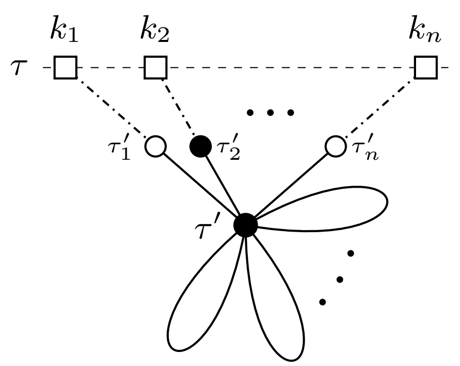
However, due to the in-in nature of the Schwinger-Keldysh formalism, we must duplicate the field φ into a time ordered field φ_+ and anti-time ordered field φ_-. Thus, two sources $J_\pm$ are required. This separation give us two type of vertices: black (time ordered) and white (anti-time ordered).

3.1.1 Feynman diagrams

Given the fact that we have translational and rotational symmetries on each time slice, we will be working in 3-momentum space[1]. In this space the diagrams will be constructed with two types of propagators:

Where a point vertex (black or white) represent a $\psi_\pm$ bulk field and a square vertex (only white) represent a ζ_+ boundary field, evaluated at specific times.

From the Lagrangian (2) and the potential (3) we know that the most general diagram up to first order (diagrams with one vertex) is:



(6)

Following the diagrammatic rules, the loops contributions appear as a integral of internal physical momentum p:

$$\sigma^2_{\text{tot}} = \frac{H^2}{4\pi^2} \int_0^\infty \frac{\mathrm{d}p}{p} \left(1 + \frac{p^2}{H^2}\right) \tag{7}$$

that is divergent.

To regularize this theory we can add counterterms to the original Lagrangian density (2), that help us to eliminate the divergent contributions coming from these *Daisy loops* shown in (6).

[1] Without transform the time coordinate

We have already done this regularization in the work [10], using on-shell and modified minimal-subtraction renormalization schemes, and using effective theory, where in the latest there is no necessity to add counterterms.

3.2 Wavefunction of the Universe

The wavefunction of the Universe formalism is based on compute a wavefunctional $\Psi[\phi]$ that is defined as the path integral of the theory action

$$\Psi[\phi(\mathbf{x})] = \int\limits_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\{iS[\Phi]\}. \tag{8}$$

Using a saddle-point approximation evaluating the action where the fields follow the classical equations of motion, we can express this functional as an exponential of a series in momentum-space

$$\begin{aligned}\Psi[\phi] &\approx \exp\{iS[\Phi_{\rm cl}]\} \\ &\equiv \exp\left\{\sum_{n=2}^{\infty}\int \frac{\mathrm{d}^3\mathbf{k}_1}{(2\pi)^3}\cdots\frac{\mathrm{d}^3\mathbf{k}_n}{(2\pi)^3}\psi_n(\mathbf{k}_1,\dots,\mathbf{k}_n)\phi(\mathbf{k}_1)\cdots\phi(\mathbf{k}_n)\right\},\end{aligned} \tag{9}$$

where $\psi_n(\mathbf{k}_1,\dots,\mathbf{k}_n)$ are the coefficients of each term of the series. These coefficients are computed in a similar way as the $n-$point functions in S-K formalism, though the propagators are different and now we have a single diagrammatic vertex. See [7] for Feynman rules of this method.

We expect that we can regularize the theory in a similar way as in our work using S-K formalism [10], with the difference that loops corrections using this formalism appear in a different form, as is shown in [11].

3.3 Bogoliubov coefficients

The third method we will use to analyze the fields statistics is by doing Bogoliubov transformation. In [12] is indicated how to write Fourier fields as a function of Bogoliubov coefficients, where these coefficients follow a first order differential equation system:

$$\frac{1}{k}\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\alpha_{\zeta a}\\ \beta_{\zeta a}\\ \alpha_{\psi a}\\ \beta_{\psi a}\end{pmatrix} = \begin{pmatrix}0 & \mathcal{P}(k\tau)\\ \mathcal{M}(k\tau) & 0\end{pmatrix}\begin{pmatrix}\alpha_{\zeta a}\\ \beta_{\zeta a}\\ \alpha_{\psi a}\\ \beta_{\psi a}\end{pmatrix}, \tag{10}$$

where $a \in \{1,2\}$, and $\mathcal{M}$ and $\mathcal{P}$ are 2×2 matrices that depend on the Fourier mode functions of the fields.

Solving this system, we can compute 2 important objects: the $2-$point function of the primordial perturbation, the power spectrum,

$$\mathcal{P}_\zeta = \frac{H^2}{4\epsilon k^3}\left(|\alpha_{\zeta 1} - \beta_{\zeta 1}|^2 + |\alpha_{\zeta 2} - \beta_{\zeta 2}|^2\right) ; \tag{11}$$

and occupation number

$$n_\zeta(k) = |\beta_{\zeta 1}|^2 + |\beta_{\zeta 2}|^2 . \tag{12}$$

None symbolic programming language can solve this problem completely. Nevertheless, we decoupled this system writing the 4th order differential equation for each coefficient, and using Laplace transformation we obtain 2nd order solvable ODEs for α_ζ and β_ζ. Now, there is only one term left to compute its inverse Laplace transformation, and we are using the convolution theorem to solve this last integral.

Also, using Maple we solve the 4th order ODE of α_ζ in function of Heun confluent functions [13] and integrals.

4 Objectives

Because we already have advanced in the tree methods that we are using in this work, now we have the following objectives to each method:

- Schwinger-Keldysh path integral formalism:
    1. Today's date, we have finished the work associated to this formalism, [10], thus there are no more objectives to achieve
- Wavefunction of the Universe:
    1. Compute the wavefunctional coefficients with Daisy loops, that are diagrams like (6)
    2. Add the corresponding counterterms to regularize the theory
- Bogoliubov coefficients:
    1. Solve the last convolution integral
    2. To have an idea of the behaviour of the solutions, solve the system numerically
    3. Find the integral constants imposing initial conditions

5 Work plan

1. **Months 1-4**: Read literature, learn S-K and Wavefunction formalism and do some well known computations to practice
2. **Months 5-25**: Computations

3. **Months 12-28**: Write the thesis
4. **Month 29**: Defend the thesis

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