# EXCITATION OF QUANTUM STATES DURING THE PRIMORDIAL UNIVERSE

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## Motivation

The Big Bang theory and inflation period predicts CMB and LSS observations as consequences of gaussian and scale-invariant perturbations in the primordial universe.

The best way of describing this phenomena is through a scalar field called inflaton. One reason to study these perturbations is that large fluctuations of $\zeta$, the curvature perturbation, can induce overdense regions with matter and produce primordial black holes (PBH) that are dark matter candidates [1]. Also, this perturbation can interact with other fields in multi-field models.

Considering more than one scalar field and solving the right action, leads to equations of motion for perturbations [2] that can be written in terms of Bogoliubov coefficients $\alpha_{Xi},\ \beta_{Xi}$. We solve numerically the equations of motion of the coefficients and we study its behavior and its impact on observable quantities as the primordial power spectrum.

## Equations of motion

To study the scalar power spectrum, and in future works GW spectrum, we define the gauge invariant quantum operators at linear order to each field [3]

$$\hat{\zeta}(\vec{k},\tau)=\frac{H}{\sqrt{2k^3}}\sum_{i=1,2}[\alpha_{\zeta i}(\tau)\zeta(k\tau)+\beta_{\zeta i}(\tau)\zeta^*(k\tau)]\hat{a}_i(\vec{k})+\text{h.c.}(-\vec{k}), \tag{1}$$

$$\hat{\psi}(\vec{k},\tau)=\frac{H}{\sqrt{2k^3}}\sum_{i=1,2}[\alpha_{\psi i}(\tau)\psi(k\tau)+\beta_{\psi i}(\tau)\psi^*(k\tau)]\hat{a}_i(\vec{k})+\text{h.c.}(-\vec{k}), \tag{2}$$

where $\zeta(k\tau)=(1+ik\tau)e^{-ik\tau}$ and $\psi(k\tau)=i\sqrt{\frac{\pi}{2}}(-k\tau)^{3/2}H_\nu^{(1)}(-k\tau)$, with $H_\nu$ the Hankel function, are the massless and massive field (with mass $\mu$) dS mode functions respectively, with $\nu=\sqrt{9/4-\mu^2/H^2}$. Both modes satisfy Bunch-Davies initial conditions at $k\tau=-\infty$.

The Bogoliubov coefficients obey the following equations of motion

$$\frac{d}{d\tau}\begin{pmatrix}\alpha_{\zeta i}\\ \beta_{\zeta i}\end{pmatrix}=A(k,\tau)\begin{pmatrix}\alpha_{\psi i}\\ \beta_{\psi i}\end{pmatrix},\qquad \frac{d}{d\tau}\begin{pmatrix}\alpha_{\psi i}\\ \beta_{\psi i}\end{pmatrix}=B(k,\tau)\begin{pmatrix}\alpha_{\zeta i}\\ \beta_{\zeta i}\end{pmatrix}, \tag{3}$$

where $i=1,2$. The initial conditions at $\tau=-\infty$ are satisfied imposing: $\alpha_{\zeta 1}=\alpha_{\psi 2}=1$ and the rest equal to zero. The coefficients matrices are given by

$$A(k,\tau)=-i\frac{\eta_\perp(\tau)}{k^3\tau^3}\begin{pmatrix}\zeta^{*\prime}\psi & \zeta^{*\prime}\psi^*\\ -\zeta^\prime\psi & -\zeta^\prime\psi^*\end{pmatrix},\qquad B(k,\tau)=i\frac{\eta_\perp(\tau)}{k^3\tau^3}\begin{pmatrix}-\zeta^\prime\psi^* & -\zeta^{*\prime}\psi^*\\ \zeta^\prime\psi & \zeta^{*\prime}\psi\end{pmatrix}, \tag{4}$$

where $\eta_\perp(\tau)$ can be any time-dependent function that represent the turns in field space, and allows us to induce the excitation of modes through the mixing of Bogoliubov coefficients.

We solve numerically these coupled differential equations in this regime for a top-hat function expressed in e-folds

$$\eta_\perp(N)=\Pi[\delta(N-N_{feat})]\cdot\gamma, \tag{5}$$

where $N_{feat}=16$, $\Pi$ is the rectangular function, $\delta$ and $\gamma$ are the width and the amplitude of the turn respectively.

Then, we compute the primordial power spectrum in terms of Bogoliubov coefficients, which is given by

$$\Delta_\zeta(k)\propto\sum_{i=1,2}\left|\alpha_{\zeta i}(k)+\beta_{\zeta i}(k)\right|^2. \tag{6}$$

## Results

To generate PBHs the scalar power spectrum must grow several orders of magnitude compared to that observed by Planck telescope, getting this we proceed computing the number of excited states and analyzing the Bogoliubov coefficients

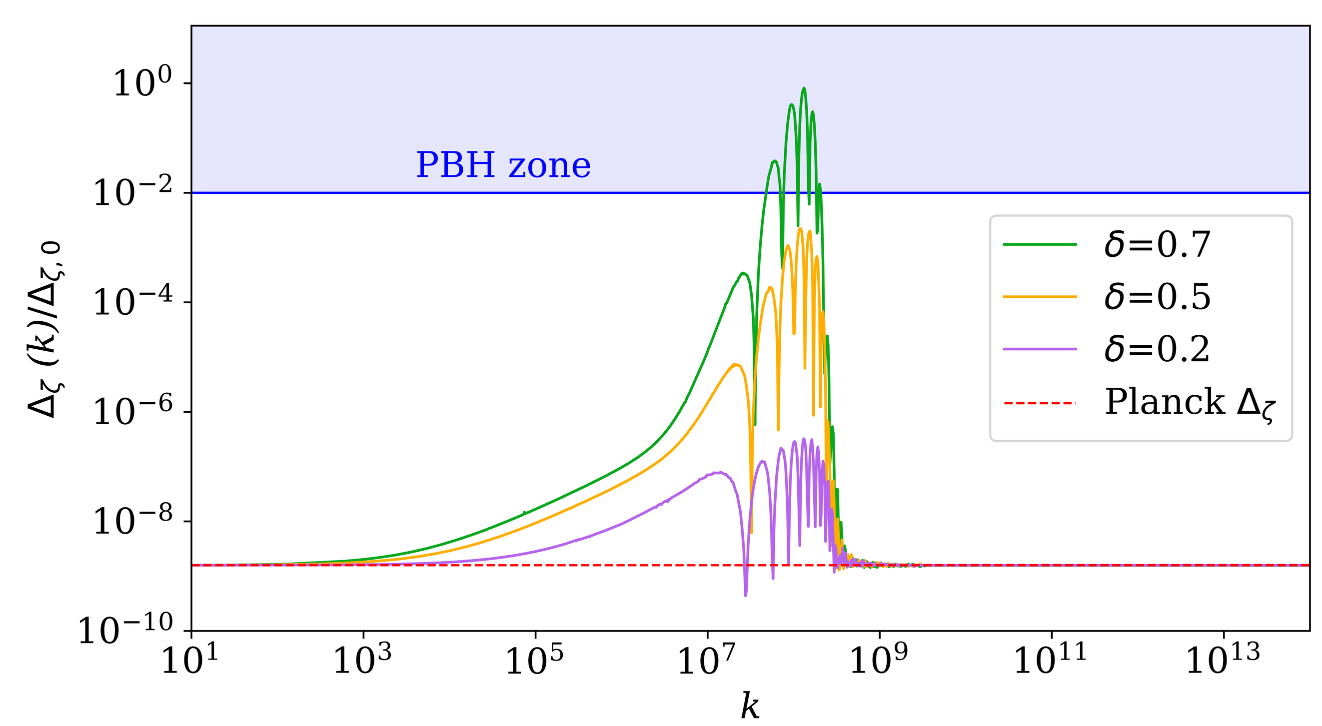


Fig. 1: Scalar power spectrum of (6) obtained using $\gamma=15$ and (3)-(4). The dashed red line show the extrapolation of $\Delta_\zeta$ observed by Planck telescope. The blue area indicate the inferior limit needed to form an interesting number of PBHs.

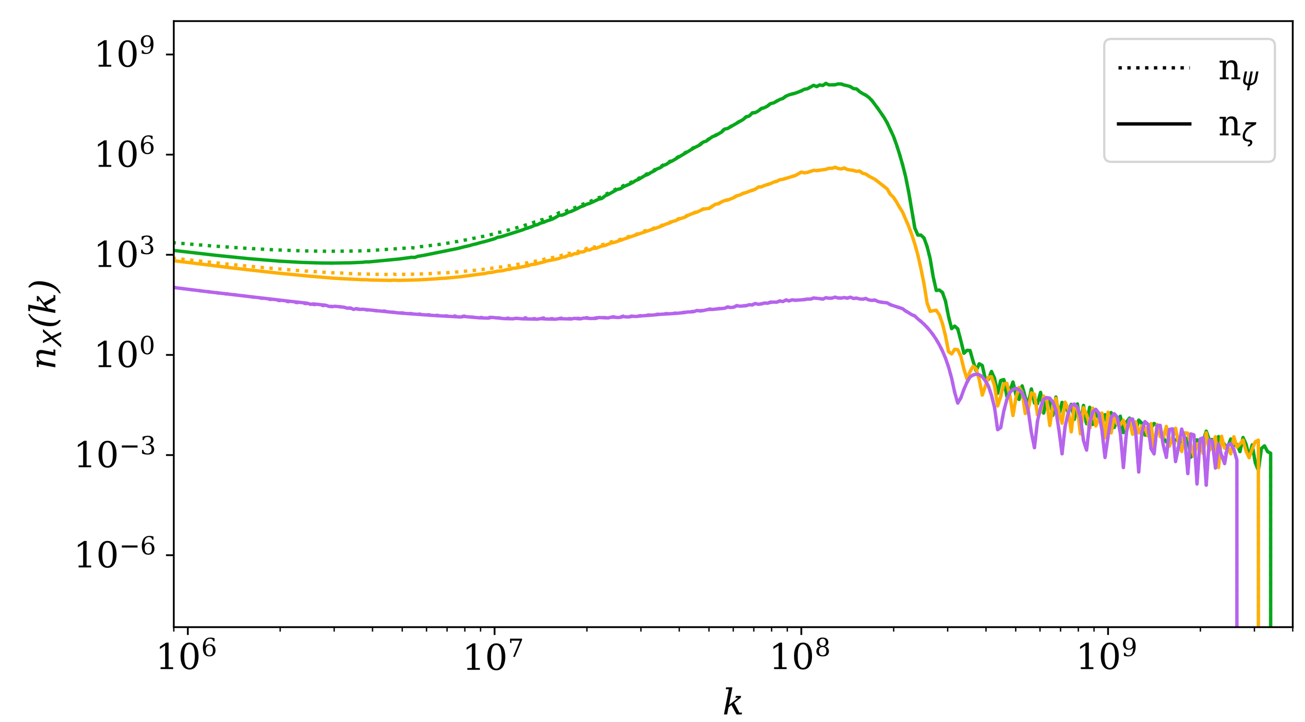


Fig. 2: Number of excited states calculated as $n_X(k)=\sum_i|\beta_{Xi}|^2$, for the same cases of Fig 1.

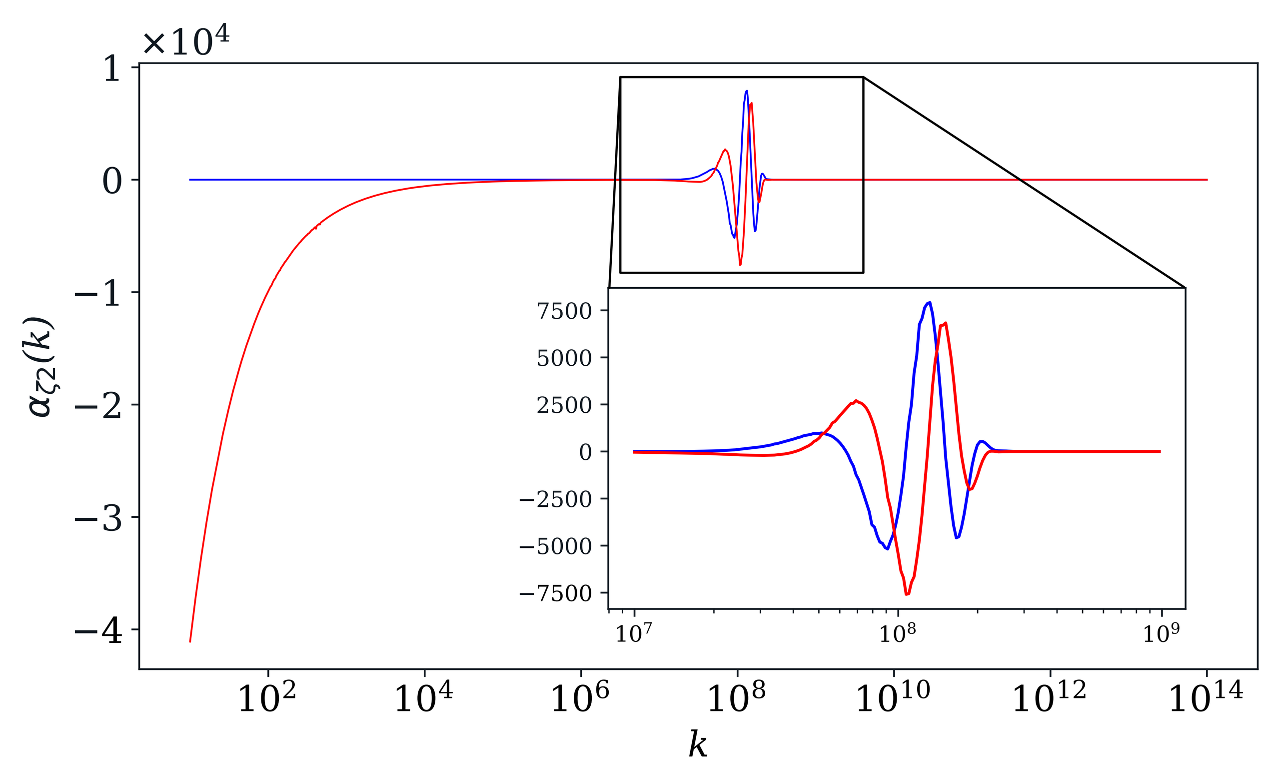


Fig. 3: Numerical solution of $\alpha_{\zeta 2}$, with the blue line the real part and the red line the imaginary part. The zoom show the oscillations present in all the coefficients, with similar shapes and magnitudes.

All the coefficients present oscillations in $k\in\{10^7,\ 10^9\}$, but the imaginary part of $\alpha,\ \beta_{\zeta 2}$ and $\alpha,\beta_{\psi 1}$ shows large tails of $\pm 4\times 10^4$.

We can not see substantial differences between $n_\zeta$ and $n_\psi$ and they follow similar behaviours as the scalar power spectrum with the same $\delta$.

The tails of the $\beta$ coefficients mentioned contribute to the number of excited states on large scales, reaching values $n_X\gtrsim 10^9$ for any value of $\delta$, but, because these coefficients at $k\approx 3\times 10^8$, $\beta\sim 1$ and for $k\approx 2\times 10^9$ suddenly takes the exact value 0, $n_X$ decrease rapidly for small scales.
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## Conclusion

Using the turning rate of the scalar fields $\eta_\perp$ to excite the quantum modes, we find that a brief, but large, turn is enough to get a $\Delta_\zeta>10^{-2}$ that, according to [4], is the limit to have a considerable number of PBH.

One way to see that is by assuming that $\zeta$ has a Gaussian distribution, so the variance [5] is calculated as

$$\sigma^2:=\int \mathrm{d}k\ k\Delta_\zeta(k), \tag{7}$$

that tell us that with a greater $\Delta_\zeta$, there are more possibilities of having larges $\zeta$ that could produce gravitational collapses and form black holes.

As we said, the spectrum of GWs, induced by the existence of excited states, can also be computed using these coefficients [3]

$$\mathcal{P}_t^{\text{out}}(k)=\int_1^\infty \mathrm{d}s\int_0^1 \mathrm{d}d\left|\alpha\left(\frac{k}{2}(s+d)\right)\right|^2\left|\alpha\left(\frac{k}{2}(s-d)\right)\right|^2F(x,y,k). \tag{8}$$

In conclusion, with the code used to get the solutions presented, we can calculate (6) and (8) to any form of $\eta_\perp$ used to excite the modes, inclusive to a single-field model (null entropy mass, $\mu=0$).

Hence, with future observations of the scalar power spectrum we can vary the parameters of the turning rate until (6) fit the data, and in this way use the Bogoliubov coefficients obtained to calculate, for example, $\mathcal{P}_t^{\text{out}}$.

The definition of the quantum operators $\hat{\zeta},\ \hat{\psi}$ as linear combinations of the Bogoliubov coefficients allow us to find, in the future or in present works, simpler expressions of other cosmology observables that could be computed numerically and, in some cases, find analytical expressions of these coefficients using fitting methods.

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## References

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