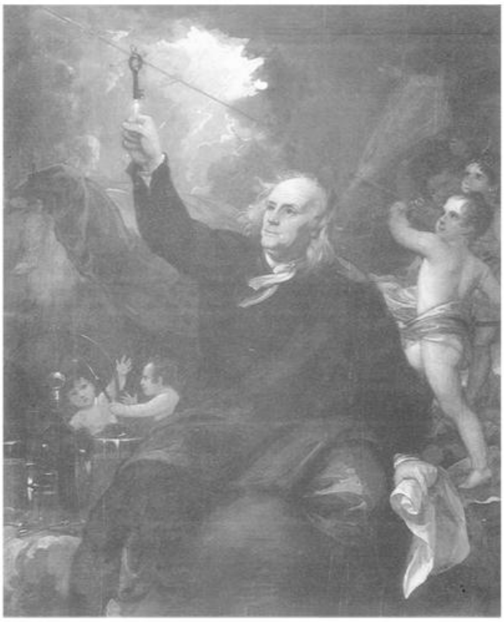
Volume 346 Issue 8 October 2009 ISSN 0016-0032

Journal of The Franklin Institute

ENGINEERING AND APPLIED MATHEMATICS



Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Journal of the Franklin Institute 346 (2009) 752–767
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Adaptive stabilization of linear and nonlinear plants in the presence of large and arbitrarily fast variations of the parameters

Manuel A. Duarte-Mermoud*, Jorge L. Estrada, Juan C. Travieso-Torres

Department of Electrical Engineering, University of Chile, Av. Tupper 2007, Casilla 412-3, Santiago, Chile

Received 3 March 2006; received in revised form 15 January 2009; accepted 22 July 2009

Abstract

The problem of adaptive stabilization of a class of continuous-time and time-varying nonlinear plants is treated in this paper. The control scheme guarantees that the state of the plant, with bounded time-varying parameters, asymptotically converges to zero. For the nonlinear case with n^2+n unknown parameters (n time-varying and n^2 constant), when the control matrix B is unknown the controller has to adjust n^2+1 parameters providing only local stability results. On the contrary, when the control matrix B is known only one parameter has to be adjusted and the proposed scheme provides global stability results. The general methodology is particularized for the linear case with $2n^2$ unknown parameters (n^2 time-varying and n^2 constant), adjusting n^2+1 parameters when the control matrix B is unknown and guarantees only local stability results, whereas in the case when the control matrix B is known only one parameter has to be adjusted and the proposed scheme provides global stability results.

Keywords: Adaptive stabilization; Adaptive control of time-varying plants; Adaptive stabilization of linear time-varying systems; Adaptive stabilization of nonlinear time-varying systems

*Corresponding author. Tel.: +56 2 978 4213; fax: +56 2 672 0162.
E-mail address: mduartem@ing.uchile.cl (M.A. Duarte-Mermoud).

0016-0032/$32.00
doi:10.1016/j.jfranklin.2009.07.004

1. Introduction

The adaptive control of time-varying linear and nonlinear plants has received considerable attention during the last two decades, particularly soon after the standard MRAC was solved in 1980 for the linear ideal case [1,2]. Several approaches have been proposed to face this problem making use of different techniques [3–8]. In this sense it is interesting to mention the work by Xie and Evans [3] on discrete time adaptive control for deterministic time-varying systems and the model reference adaptive control system proposed by Ohkawa [4] for discrete linear time-varying systems with periodically varying parameters and time-delay. Middleton and Goodwin [5] suggested a global adaptive control scheme for time-varying linear systems, based on bounded-input bounded-state stability without requiring persistent excitation. An indirect adaptive control scheme was also proposed by Tsakalis and Ioannou [6], for time-varying plants whose parameter variations are not necessarily slow. Marino and Tomei [7] presented an adaptive output feedback control for a class of nonlinear SISO observable, minimum phase systems with unknown time-varying parameters belonging to a known compact set whose time derivatives are bounded, but are not restricted to be small or to have known bounds. More recently, Ge and Wang [8] proposed a robust adaptive tracking method for time-varying nonlinear systems in the strict feedback form with completely unknown time-varying virtual control coefficients, uncertain time-varying parameters and unknown time-varying bounded disturbances. The proposed design method does not require any a priori knowledge of the unknown coefficients except for their bounds.

Also, in the area of chaos control, initiated by Ott et al. [22] for the case of known parameters, when the parameters are unknown but constant, different techniques have been used [9–15,18,19], but when the parameters are unknown and time-varying, only few results have been reported [16,17]. The techniques used in the case when the parameters are unknown but constant include adaptive observers, proposed by Liao and Tsai [10], adaptive backstepping [9,14,15,18], Lyapunov's stability theory [11–13] and theory of invariant manifolds [19].

In the case of time-varying uncertain chaotic systems Li et al. [16] proposed a robust adaptive tracking control for a class of nonlinear plants when the control matrix is known and equal to the identity. The desired trajectory and its first time derivative are assumed to be known. The method imposes two assumptions on the plant to be controlled, the second one being very restrictive. This method was simplified in [17] relaxing the second assumption and assuming that a desired trajectory and its first time derivative are known to the designer. Later, in [21], a method where the constraint of the second assumption is moved from the plant to the reference model introduced, is presented. Lately, a further attempt to generalize these results was made in [20] where it is considered only one assumption concerning the boundedness of the time-varying parameters and both cases when a model reference or a desired trajectory and its time derivative are known, were resolved.

In this paper a new effort to generalize these previous results is made, considering the adaptive stabilization of a class of nonlinear plants with arbitrarily fast time-variations, when the control matrix B is unknown but constant and boundedness on the time-varying parameters is the only assumption.

The paper is organized as follows. In Section 2, the adaptive stabilization of a class of nonlinear plants is treated, considering the case when the control matrix B is both unknown and known. Section 3 is devoted to the adaptive stabilization when the plant is linear, discussing the cases of B unknown and known. In Section 4, simulation results for the case of nonlinear and linear plants are presented, considering only the case of B unknown. Finally, in Section 5 some conclusions are drawn.

2. Adaptive stabilization of nonlinear plants

Let us consider the nonlinear plant described by the following dynamics

$$\dot{x}(t) = f(x) + F(x)\theta(t) + Bu(t) \tag{1}$$

where $x \in \Re^n$ corresponds to the state vector of the system, which is assumed to be accessible, $f(x) \in \Re^n$ and $F(x) \in \Re^{n\times p}$ are known continuously differentiable functions with $f(0) = 0$ and $F(0) = 0$. $f(x)$ and $F(x)$ are bounded for x bounded. $B \in \Re^{n\times n}$ is a constant, nonsingular but unknown matrix, $u(t) \in \Re^n$ is the input to the plant and $\theta(t) \in \Re^p$ is the unknown time-varying parameter vector, which we assume belongs to a bounded and closed set, as stated in the following assumption.

Assumption 1. The unknown parameter vector $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]^T \in \Re^p$ belongs to a bounded and closed set Ω, where $\Omega = [\underline{\theta}_1, \bar{\theta}_1] \times [\underline{\theta}_2, \bar{\theta}_2] \times \cdots \times [\underline{\theta}_p, \bar{\theta}_p]$, with $\underline{\theta}_i, \bar{\theta}_i$ for $i = 1, 2, \ldots, p$ unknown constants representing the lower and upper bounds, respectively, on the components of vector $\theta(t) \in \Re^p$.

From Assumption 1, we can immediately write

$$\|\theta(t)\| = \left(\sum_{i=1}^{p} \theta_i(t)^2\right)^{1/2} \leq \left(\sum_{i=1}^{p} \max[|\underline{\theta}_i|^2, |\bar{\theta}_i|^2]\right)^{1/2} \triangleq \beta \tag{2}$$

where $\beta \in \Re$ is an unknown constant parameter.

The objective is to determine a bounded input $u(t)$ such that $x(t)$ goes to zero asymptotically, in spite of the arbitrarily fast time variations of the plant parameters. In particular we are interested that

$$\lim_{t\to\infty} x(t) = 0 \tag{3}$$

either globally or in a certain region around the origin. We express $f(x) \in \Re^n$ function in terms of $f'(x) \in \Re^n$ function as

$$f(x) = A_m x(t) + f'(x) \tag{4}$$

with

$$f'(x) = f(x) - A_m x(t) \tag{5}$$

where $A_m \in \Re^{n\times n}$ is any asymptotically stable matrix. We then choose the control law $u(t) \in \Re^n$ in the following fashion

$$u(t) = K_1(t)[-f'(x) + \alpha(e, x, \widehat{\beta})] \tag{6}$$

where $K_1(t) \in \Re^{n\times n}$ is a matrix of adjustable parameters and $\alpha(e, x, \widehat{\beta}) \in \Re^n$ is given by

$$\alpha(x, \widehat{\beta}) = -\frac{F(x)\mu(x)\widehat{\beta}^{\,2}}{\|\mu(x)\|\,\widehat{\beta} + \varepsilon\|x\|^2} \tag{7}$$

with

$$\mu^T(x) = x^T P F(x) \tag{8}$$

where $P = P^T \in \Re^{n \times n}$ is a symmetric and positive definite matrix solution of the Lyapunov equation

$$A_m^T P + P A_m = -Q \tag{9}$$

where $Q = Q^T \in \Re^{n \times n}$ is any symmetric and positive definite matrix, with $0 < 2\varepsilon < \lambda_{\min}(Q)$, where $\lambda_{\min}(Q)$ is the minimum eigenvalue of matrix Q. We also choose the adaptive law for$\beta(t) \in \Re$ given by

$$\dot{\widehat{\beta}}(t) = \gamma \|\mu(x)\| \tag{10}$$

with $\widehat{\beta}(t_0) > 0$ and $\gamma > 0$. Then we can state the following theorem regarding the stability of the adaptive system (1), (5)–(10).

Theorem 1. *Let us suppose that* Assumption 1 *is satisfied for system* (1). *If we choose the control law given by Eqs.* (5)–(9) *with* $0 < 2\varepsilon < \lambda_{\min}(Q)$ *and the adaptive law given by Eq.* (10) *with* $\widehat{\beta}(t_0) > 0$ and $\gamma > 0$, *then the resulting system is locally uniformly stable and* $\lim_{t\to\infty} x(t) = 0$, *provided the parameters* $K_1(t)$ *is adjusted in the following fashion:*

$$\dot{K}_1 = K_1 P x [f'(x) - \alpha(x, \hat{\beta})]^T K_1^T K_1 \tag{11}$$

It is important to notice that since

$$\frac{d}{dt}(K_1^{-1}(t)) = -K_1(t)\left(\frac{d}{dt}K_1(t)\right)K_1(t) \tag{12}$$

the adaptive law (11) can be written in an equivalent form as

$$\dot{\Phi}_{K_1}(t) = -Px[f'(x) - \alpha(x, \hat{\beta})]^T K_1^T(t) \tag{13}$$

where $\Phi_{K_1}(t) \in \Re^{n \times n}$ is defined as

$$\Phi_{K_1}(t) = K_1^{-1}(t) - K_1^{*-1} \tag{14}$$

with $K_1{*}^{-1} \in \Re^{n \times n}$ defined in Eq. (18).

Proof. Replacing Eq. (6) in Eq. (1) we can write

$$\dot{x}(t) = A_m x(t) + f'(x) - BK_1 f'(x) + F(x)\theta(t) + BK_1 \alpha(x, \widehat{\beta}) \tag{15}$$

Adding and subtracting the term $\alpha(x, \widehat{\beta})$to Eq. (15), the dynamics of the state is given by

$$\dot{x}(t) = A_m x(t) + [I - BK_1]f'(x) - [I - BK_1]\alpha(x, \widehat{\beta}) + F(x)\theta(t) + \alpha(x, \widehat{\beta}) \tag{16}$$

Regrouping terms we get

$$\dot{x}(t) = A_m x(t) + [I - BK_1][f'(x) - \alpha(x, \widehat{\beta})] + F(x)\theta(t) + \alpha(x, \widehat{\beta}) \tag{17}$$

If we define the following ideal controller parameters $K{*}_1 \in \Re^{n \times n}$ satisfying the following equation

$$[I - BK_1^*] = 0 \Rightarrow K_1^* = B^{-1} \text{ or } K_1^{*-1} = B \tag{18}$$

we can write the following identity:

$$[I - BK_1] = [K_1^{-1} - B]K_1 = [K_1^{-1} - K_1^{*-1}]K_1 = \Phi_{K_1} K_1 \tag{19}$$

$$[I - BK_1(t)] = \Phi_{K_1}(t)K_1(t) \tag{20}$$

with $\Phi_{K_1} \in \Re^{n\times n}$ defined in Eq. (14).

Then we can rewrite Eq. (17) as

$$\dot{x}(t) = A_m x(t) + \Phi_{K_1} K_1[f'(x) - \alpha(x, \widehat{\beta})] + F(x)\theta(t) + \alpha(x, \widehat{\beta}) \tag{21}$$

Now we will analyze the stability of the resulting adaptive system defined by Eqs. (21), (10) and (11) (or Eq. (13)). To that extent we choose the following Lyapunov function candidate

$$V(x, \Phi_{K_1}, \tilde{\beta}) = \frac{1}{2}\left(x^T P x + Trace\{\Phi_{K_1}\Phi_{K_1}^T\} + \frac{1}{\gamma}\tilde{\beta}^2\right) \tag{22}$$

where $x(t) \in \Re^n$, $\Phi_{K_1}(t) = K_1^{-1}(t) - K_1^{*-1} \in \Re^{n\times n}$, with $K_1^{*-1} \in \Re^{n\times n}$ defined in Eq. (18), and $\tilde{\beta}(t) = \beta(t) - \beta \in \Re$, with β defined by Eq. (2).

We now compute the first time derivative of Eq. (22) along the system (21), (10) and (13)

$$\dot{V} = \frac{1}{2}\left(\dot{x}^T P x + x^T P \dot{x} + 2Trace\{\dot{\Phi}_{K_1}\Phi_{K_1}^T\} + \frac{2}{\gamma}\tilde{\beta}\dot{\tilde{\beta}}\right) \tag{23}$$

Replacing $\dot{x}^T$ and $\dot{x}$ given by Eq. (21) in the previous expression we get

$$\begin{aligned}\dot{V} = \frac{1}{2}(&x^T A_m^T P x + [f'(x) - \alpha(x, \widehat{\beta})]^T K_1^T \Phi_{K_1}^T P x \\ &+ \theta^T F^T(x) P x + \alpha^T(x, \widehat{\beta}) P x + x^T P A_m x \\ &+ x^T P \Phi_{K_1} K_1[f'(x) - \alpha(x, \widehat{\beta})] + x^T P F(x)\theta \\ &+ x^T P \alpha(e, x, \widehat{\beta}) + 2Trace\{\dot{\Phi}_{K_1}\Phi_{K_1}^T\} + \frac{2}{\gamma}\tilde{\beta}\dot{\tilde{\beta}})\end{aligned} \tag{24}$$

Regrouping terms and using the property of two vectors $a \in \Re^n$ and $b \in \Re^n$ that $a^T b = b^T a = Trace\{ab^T\} = Trace\{ba^T\}$ we get

$$\begin{aligned}\dot{V} = &\frac{1}{2}x^T(A_m^T P + PA_m)x + Trace\{Px[f'(x) - \alpha(x, \widehat{\beta})]^T K_1^T \Phi_{K_1}^T\} \\ &+ x^T P F(x)\theta + x^T P\alpha(e, x, \widehat{\beta}) + Trace\{\dot{\Phi}_{K_1}\Phi_{K_1}^T\} + \frac{1}{\gamma}\tilde{\beta}\dot{\tilde{\beta}}\end{aligned} \tag{25}$$

Replacing the adaptive law given by Eq. (13) in Eq. (25) we obtain

$$\dot{V} = -\frac{1}{2}x^T Q x + x^T P F(x)\theta + x^T P\alpha(x, \widehat{\beta}) + \frac{1}{\gamma}\tilde{\beta}\dot{\tilde{\beta}} \tag{26}$$

where Q is the matrix chosen in the Lyapunov equation (9). Replacing the definition of $\alpha(x, \widehat{\beta})$ and $\mu(x)$ given by Eqs. (7) and (8), respectively, in Eq. (26) we get

$$\dot{V} = -\frac{1}{2}x^T Q x + \mu^T(x)\theta(t) - \frac{\|\mu(x)\|^2 \widehat{\beta}^2}{\|\mu(x)\|\,\widehat{\beta} + \varepsilon\|x\|^2} + \frac{1}{\gamma}\tilde{\beta}\dot{\tilde{\beta}} \tag{27}$$

From Assumption 1 and Eq. (2) we can write the following inequality

$$\mu^T(x)\theta(t) \leq \|\mu(x)\|\beta \tag{28}$$

Moreover, the following inequality can be established

$$\begin{aligned}-\frac{\|\mu(x)\|^2\widehat{\beta}^2}{\|\mu(x)\|\,\widehat{\beta}+\varepsilon\|x\|^2} &= \|\mu(x)\|\,\widehat{\beta}\left(-1+\frac{\varepsilon\|x\|^2}{\|\mu(x)\|\,\widehat{\beta}+\varepsilon\|x\|^2}\right)\\ &\leq \|\mu(x)\|\,\widehat{\beta}\left(-1+\frac{\varepsilon\|x\|^2}{\|\mu(x)\|\,\widehat{\beta}}\right)\end{aligned} \tag{29}$$

Furthermore, it is easy to verify that

$$-\frac{1}{2}x^TQx \leq -\frac{\lambda_{\min}(Q)}{2}\|x\|^2 \tag{30}$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of the positive definite matrix Q.

Replacing Eqs. (28)–(30) in Eq. (27) we get

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q)\|x^2\| + \|\mu(x)\|\beta + \|\mu(x)\|\,\widehat{\beta}\left(-1+\frac{\varepsilon\|x\|^2}{\|\mu(x)\|\,\widehat{\beta}}\right) + \frac{1}{\gamma}\tilde{\beta}\dot{\widehat{\beta}} \tag{31}$$

which can be rewritten as

$$\dot{V} \leq -\left(\frac{1}{2}\lambda_{\min}(Q)-\varepsilon\right)\|x^2\| - \|\mu(x)\|\tilde{\beta} + \frac{1}{\gamma}\tilde{\beta}\dot{\widehat{\beta}} \tag{32}$$

Finally, replacing the adaptive law given by Eq. (10) in Eq. (32) we obtain

$$\dot{V} \leq -\left(\frac{1}{2}\lambda_{\min}(Q)-\varepsilon\right)\|x^2\| \tag{34}$$

Since $0<2\varepsilon<\lambda_{\min}(Q)$ then $\dot{V}\leq 0$. Therefore $x(t)$, $\Phi_{K_1}(t)$ and $\tilde{\beta}(t)$ are globally uniformly bounded. From this we can conclude that $\beta(t)$ is also globally uniformly bounded. From the definition of $\Phi_{K_1}(t)$ given in Eq. (14) we can conclude that $K_1^{-1}(t)\in\Re^{n\times n}$ is globally uniformly bounded but $K_1(t)\in\Re^{n\times n}$ is only locally uniformly bounded. From Eq. (34) we can conclude that $x(t)$ is a signal of square integral. From Eqs. (6)–(8) it follows that the control signal $u(t)$ is locally uniformly bounded. Consequently, from Eq. (21) we conclude that $\dot{x}(t)$ is locally uniformly bounded, since it corresponds to a sum and products of locally uniformly bounded functions. Using the Lemma of Barbalat [24], we can conclude that locally, $x(t)\to 0$ when $t\to\infty$. Therefore, the controller given by Eqs. (6)–(8) and the adaptive laws given by Eqs. (10) and (11) (or Eq. (13)) guarantee that system (1) is locally uniformly stable. □

Remark 1. Notice that in the previous development (Theorem 1) for the general case of B unknown we need to adjust n^2+1 parameters to control plant (1) having n^2+n unknown parameters (n time-varying and n^2 constant) and provides only local stability results.

Remark 2. In the previous analysis done in Section 2, unity adaptive gains were chosen for simplicity in all the adaptive laws (10) and (11) (or (13)) used in the design. It is possible to show that all the results stated in Section 2 will also be valid if constant and positive scalars adaptive gains are used, or constant and positive definite matrices adaptive gains are introduced, or finally, time-varying matrices adaptive gains with a special type of variation are defined [25,26]. The effect of these adaptive gains will be to improve the transient behavior of the resultant adaptive system.

Remark 3. The convergence of the controller parameter is not guaranteed in the proposed control scheme. This is achieved only if persistently exciting conditions are met for the vectors and matrices involved in the adaptive laws (10) and (11) (or (13)).

Remark 4. If the control matrix B has certain particular form, the structure of the proposed control scheme can be simplified and the scope of the method can be enlarged. For example if B is a diagonal matrix, invertible, and the sign of all elements on the diagonal are known, then the resultant controller and adaptive laws have the following form [23,26]

$$u(t) = K_1(t)[-f'(x) + \alpha(e, x, \widehat{\beta})]$$

with

$$\dot{K}_1(t) = sign\{B\}Px[f'(x) - \alpha(x, \hat{\beta})]^T$$

where for notation purposes we have $B = sign\{B\}|B|$. In this case the results are proven to be global rather than local [23,26]. Same kind of simplifications can be obtained if the matrix B is positive definite (and invertible) [23,26] obtaining again global stability results. Finally, when the matrix B has any general form then we get the results shown in Section 2, which are only local in nature.

Remark 5. When the matrix B is known, the ideal controller parameter $K_1^* \in \Re^{n \times n}$ given by Eq. (18) is also known and can be computed and replaced in the control law (6) becoming

$$u(t) = B^{-1}[-f'(x) + \alpha(e, x, \widehat{\beta})]$$

with $\alpha(x, \widehat{\beta}) \in \Re^n$ and $\mu(x) \in \Re^p$ given by Eqs. (7) and (8) respectively. In the previous case, when B is known, adaptation for $K_1(t) \in \Re^{n \times n}$ is not needed. Thus, uniform global stability (instead of local) can be achieved for the adaptive system adjusting only the parameter $\beta \in \Re$, with the adaptation given by Eq. (10) (see Table 1).

Remark 6. When the matrix B is known, no additional assumptions to the boundedness of time-varying parameters are needed in the approach proposed here, whereas in the method presented by Li et al. [16] a second quite restrictive assumption is stated, which narrows the class of nonlinear systems to which the methodology can be applied. Besides, the desired trajectory and its first time derivative are assumed to be known. This method (for B known) was simplified in [17] relaxing the second assumption and assuming that a desired trajectory and its first time derivative are known to the designer. Later, in [21] a method is presented where the constraint of the second assumption is moved from the plant to the reference model. Recently, an attempt to generalize the results in [16,17,21] for the case of B known was made in [20], where it is considered only the assumption on the boundedness of the time-varying parameters and both cases when a model reference or a desired trajectory and its time derivative are known, were resolved.

Table 1
Summary of stability results.

B	$f(x)$	$F(x)\theta(t)$	$u(t)$	$\alpha(x,\widehat{\beta})$	$\mu(x)$	Adaptive laws	Stability
B $B \in \Re^{n\times n}$	$f(x)$ $f(x) \in \Re^n$	$F(x)\theta(\mathrm{t})$ $\theta(t) \in \Re^p$ $F(x) \in \Re^{n\times p}$	$-K_1(t)[f'(x) - \alpha(e,x,\widehat{\beta})]$ $\widehat{\beta} \in \Re,\ \ K_1(t) \in \Re^{n\times n}$	$-F(x)\mu(x)\widehat{\beta}^2/\lVert\mu(x)\rVert\,\widehat{\beta}+\varepsilon\lVert x\rVert^2$	$F^T(x)Px$ $\mu \in \Re^n$ ${A_m}^T P + PA_m = -Q$	$\dot{\widehat{\beta}}(t) = \gamma\lVert\mu(x)\rVert$ $\dot{K}_1 = K_1 Px[f'(x) - \alpha(x,\widehat{\beta}]^T K_1^T K_1$	Local
B known $B \in \Re^{n\times n}$	$f(x)$ $f(x) \in \Re^n$	$F(x)\theta(\mathrm{t})$ $\theta(t) \in \Re^p$ $F(x) \in \Re^{n\times p}$	$-B^{-1}[f(x) - A_m x(t) + \alpha(e,x,\widehat{\beta})]$ $\widehat{\beta} \in \Re$	$-F(x)\mu(x)\widehat{\beta}^2/\lVert\mu(x)\rVert\,\widehat{\beta}+\varepsilon\lVert x\rVert^2$	$F^T(x)Px$ $\mu \in \Re^n$ ${A_m}^T P + PA_m = -Q$	$\dot{\widehat{\beta}}(t) = \gamma\lVert\mu(x)\rVert$	Global
B $B \in \Re^{n\times n}$	$f(x)$ $f(x) \in \Re^n$	$\Theta(t)x(t)$ $\Theta(t) \in \Re^{n\times n}$ $x(t) \in \Re^n$	$K(t)\alpha(x,\widehat{\beta})$ $\widehat{\beta} \in \Re,\quad K(t) \in \Re^{n\times n}$	$-\lVert x\rVert^2 Px\widehat{\beta}^2/\lVert\mu(x)\rVert_F\,\widehat{\beta}+\varepsilon\lVert x\rVert^2$	$xx^T P$ $\mu \in \Re^{n\times n}$ ${A_m}^T P + PA_m = -Q$	$\dot{\widehat{\beta}}(t) = \gamma\lVert\mu(x)\rVert_F$ $\dot{K}(t) = -KPxu^T K$	Local
B known $B \in \Re^{n\times n}$	$f(x)$ $f(x) \in \Re^n$	$\Theta(t)x(t)$ $\Theta(t) \in \Re^{n\times n}$ $x(t) \in \Re^n$	$B^{-1}\alpha(x,\widehat{\beta})$ $\widehat{\beta} \in \Re^n$	$-\lVert x\rVert^2 Px\widehat{\beta}^2/\lVert\mu(x)\rVert_F\,\widehat{\beta}+\varepsilon\lVert x\rVert^2$	$xx^T P$ $\mu \in \Re^{n\times n}$ ${A_m}^T P + PA_m = -Q$	$\dot{\widehat{\beta}}(t) = \gamma\lVert\mu(x)\rVert_F$	Global

3. The linear case

Based on the results shown in Section 2, several particular cases can be derived. Because of its general interest we will study in detail the case when the plant is time-varying and linear with accessible state defined by the following differential equation

$$\dot{x}(t) = \Theta(t)x(t) + Bu(t) \tag{35}$$

where $x(t) \in \Re^n$ is the state of the system, $\Theta(t) \in \Re^{n\times n}$ represents the matrix of time-varying and unknown parameters and the matrix $B \in \Re^{n\times n}$ is a nonsingular matrix of unknown but constant parameters. $u(t) \in \Re^n$ is the plant input. It is assumed that time-varying elements of the matrix parameter $\Theta(t)$ are bounded, as stated in the following assumption (equivalent to Assumption 1 for the vector case).

Assumption 2. The matrix $\Theta(t) \in \Re^{n\times n}$, belongs to a bounded and closed set Ω defined by $\Omega = \{\Theta(t) = [\theta_{ij}(t)] \in \Re^{n\times n} / \theta_{ij}(t) \in [\underline{\theta}_{ij}, \bar{\theta}_{ij}]\}$, with $\underline{\theta}_{ij}, \bar{\theta}_{ij}$ for $i,j = 1,2,\ldots,n$ unknown constant parameters representing the lower and upper bounds, respectively, on the time-varying parameters $\theta_{ij}(t)$, the elements of matrix $\Theta(t)$.

The plant given in Eq. (35) can be rewritten as follows:

$$\dot{x}(t) = A_m x(t) + (\Theta(t) - A_m)x(t) + Bu(t) \tag{36}$$

where $A_m \in \Re^{n\times n}$ is any asymptotically stable matrix.

We define the unknown matrix with time-varying parameters $\bar{A}(t) = [\bar{a}_{ij}(t)] \in \Re^{n\times n}$ as

$$\bar{A}(t) = \Theta(t) - A_m \in \Re^{nxn} \tag{37}$$

From Assumption 2, we can write the following inequality

$$\begin{aligned} \|\bar{A}(t)\|_F = (Tr\{\bar{A}^T\bar{A}\})^{1/2} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\bar{a}_{ij}^2(t)\right)^{1/2} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}(\theta_{ij}(t) - a_{ij})^2\right)^{1/2} \\ \le \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\max\{|\underline{\theta}_{ij} - a_{ij}|^2, |\bar{\theta}_{ij} - a_{ij_i}|^2\}\right)^{1/2} \triangleq \beta \end{aligned} \tag{38}$$

where $\beta \in \Re$ is an unknown constant parameter. $\|\cdot\|_F$ denotes the Frobenius norm of a matrix.

Now we can state the following theorem for adaptively control system (35).

Theorem 2. Let us consider the linear time-varying system defined by Eq. (35). Let us assume that Assumption 2 is satisfied and define the control law as

$$u(t) = K(t)\alpha(x, \widehat{\beta}) \tag{39}$$

where $K(t) \in \Re^{n\times n}$ is an adjustable parameter and $\alpha(x, \widehat{\beta}) \in \Re^n$, $\mu(x) \in \Re^{n\times n}$ are given by

$$\alpha(x, \widehat{\beta}) = -\frac{\|x\|^2 Px\widehat{\beta}^2}{\|\mu(x)\|_F\widehat{\beta} + \varepsilon\|x\|^2} \tag{40}$$

$$\mu(x) = xx^T P \tag{41}$$

with $0<2\varepsilon<\lambda_{\min}(Q)$ and $P=P^T\in\Re^{n\times n}$ solution of Eq. (9). Let us consider the adaptive law for $\beta(t)$ given by

$$\dot{\widehat{\beta}}(t)=\gamma\|\mu(x)\|_F \tag{42}$$

with $\widehat{\beta}(t_0)>0$ and $\gamma>0$ an adaptive gain, together with the adaptive law for $K(t)\in\Re^{n\times n}$ (the estimate of $K^*=B^{-1}\in\Re^{n\times n}$)

$$\dot{K}(t)=-K(t)B_m^T Pxu^T K(t) \tag{43}$$

Then, the overall adaptive system is locally uniformly stable and also the state of the linear time-varying system (35) will asymptotically converge to zero.

It is important to notice that the adaptive law (43) can be written equivalently using property (12) as follows:

$$\dot{\Phi}_K(t)=Pxu^T \tag{44}$$

where

$$\Phi_K(t)=K^{-1}(t)-K^{*-1} \text{ with } K^{*-1}=B\in\Re^{n\times n} \tag{45}$$

Proof. The proof follows along the same lines as in Theorem 1 for the nonlinear case. Here we choose the following Lyapunov function candidate

$$V(x,\Phi_K,\tilde{\beta})=\frac{1}{2}\left(x^T Px+Trace\{\Phi_K\Phi_K^T\}+\frac{1}{\gamma}\tilde{\beta}^2\right) \tag{46}$$

where $\tilde{\beta}(t)=\widehat{\beta}(t)-\beta\in\Re$ and $\Phi_K(t)=K^{-1}(t)-K^{*-1}\in\Re^{n\times n}$. Computing the first derivative of Eq. (46) we end up with

$$\dot{V}\leq-\left(\frac{1}{2}\lambda_{\min}(Q)-\varepsilon\right)\|x\|^2 \tag{47}$$

Since $0<2\varepsilon<\lambda_{\min}(Q)$ then $\dot{V}\leq 0$ and the overall system is globally uniformly stable. In particular $x(t)$, $\Phi_K(t)$ and $\tilde{\beta}(t)$are globally uniformly bounded. From this we can conclude that $\beta(t)$ is also globally uniformly bounded. From the definition of $\Phi_K(t)$ given in Eq. (45) we can conclude that $K^{-1}(t)\in\Re^{n\times n}$ is globally uniformly bounded but $K(t)\in\Re^{n\times n}$ is only locally uniformly bounded. From Eq. (47) we can conclude that $x(t)$ is a signal of square integral ($x(t)\in L^2$). From Eqs. (39)–(41) it follows that the control signal $u(t)$ is locally uniformly bounded. Consequently, from Eqs. (35), (37), (39) and (40) we conclude that $\dot{x}(t)$ is locally uniformly bounded ($\dot{x}(t)\in L^\infty$), since it corresponds to a sum and products of locally uniformly bounded functions. Using the Lemma of Barbalat [24], we can conclude that locally, $x(t)\to 0$ when $t\to\infty$. Therefore, the controller given by Eqs. (39)–(41) and the adaptive laws given by Eqs. (42) and (43) (or Eq. (44)) guarantee that the system (35) is locally uniformly stable. □

Remark 7. Theorem 2, applicable to linear systems with $2n^2$ unknown parameters (n^2 time-varying and n^2 constant) of the form (37), guarantees that locally all the signals remain bounded and the state asymptotically converges to zero by adjusting n^2+1 parameters, providing local uniform stability results for the general case of B unknown.

Remark 8. In the previous analysis done in Section 3, unity adaptive gains were chosen for simplicity in all the adaptive laws (42) and (43) (or (44)) used in the design. It is possible to show that all the results stated in Section 2 will also be valid if constant and positive scalars adaptive gains are used, or constant and positive definite matrices adaptive gains are introduced, or finally, time-varying matrices adaptive gains with a special type of variation are defined [25,26]. The effect of these adaptive gains will be to improve the transient behavior of the resultant adaptive system.

Remark 9. The convergence of the controller parameter is not guaranteed in the proposed control scheme. This is achieved only if persistently exciting conditions are met for the vectors and matrices involved in the adaptive laws (42) and (44).

Remark 10. If the control matrix B has certain particular form, the structure of the proposed control scheme can be simplified and the scope of the method can be enlarged, as in the nonlinear case. For example if B is a diagonal matrix, invertible, and the sign of all elements on the diagonal are known, then the resultant controller and adaptive laws have the following form [23,26]

$$u(t) = K(t)\alpha(x, \widehat{\beta})$$

with

$$\dot{K}(t) = sign\{B\}Pxu^T$$

where for notation purposes we have $B = sign\{B\}|B|$. In this case the results are proven to be global rather that local [23,26]. Same kind of simplifications can be obtained if the matrix B is positive definite and invertible [23,26] obtaining again global stability results. Finally, when the matrix B has any general form then we get the results shown in Section 3, which are only local in nature.

Remark 11. For the case when the matrix B is known, it can be shown that the resulting adaptive scheme adjusts only one parameter. In fact, since B is known so is $K^* = B^{-1}B_m \in \Re^{n\times n}$. Replacing $K(t)$ in the control law (39) by the ideal controller parameter K^* the control input becomes

$$u(t) = B^{-1}\alpha(x, \widehat{\beta}) \tag{48}$$

with $\alpha(x, \widehat{\beta}) \in \Re^n$ and $\mu(x) \in \Re^p$ given by Eqs. (44) and (45), respectively. Therefore, adaptation for $K(t) \in \Re^{n\times n}$ is not needed. Thus, uniform global stability can be achieved for the adaptive system adjusting only the parameter $\widehat{\beta} \in \Re$, with the adaptation given by Eq. (42) (see Table 1).

4. Simulation results

In order to verify the behavior of the proposed method we will simulate the case of a nonlinear plant in Section 4.1 and for the case of a linear plant in Section 4.2.

4.1. Nonlinear system

Let us consider the second order continuous-time nonlinear plant of the form (1) defined by

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1x_2^2\theta_1 + b_1u_1 \\ \dot{x}_2 &= x_1x_2 + x_2x_1^2\theta_2 + b_2u_2 \end{aligned} \tag{49}$$

where $\theta_1(t) = 2(1+0.1\cos(1.5t))$, $\theta_2(t) = 4(1+0.15\cos(0.75t))$, $b_1 = 1.8$ and $b_2 = 3.2$. Beside, the following initial conditions were chosen $x(0) = [0.7 \quad 0.5]^T \in \Re^2$.

The controller $u(t) \in \Re^2$ in this case is defined by Eqs. (6)–(9), together with the adaptive laws (10) and (11). The controller parameters were chosen as $\varepsilon = 10$ and $\gamma = 2$, and the initial conditions for the adaptive parameters have the following numerical values $\beta(0) = 1$, $K_1(0) = \begin{bmatrix} 0.50 & 0.03 \\ 0.02 & 0.30 \end{bmatrix}$.

Fig. 1 shows the evolution of the states of the plant. It can be observed that the state goes to zero as t goes to infinity. The behavior of the controller parameters $\beta(t)$ and $K_1(t)$ are shown in Fig. 2. As expected, the parameter $\beta(t)$ evolves from its initial value up to a final bounded value determined by the stability conditions of the adaptive system. Finally, the evolution of the control input applied to the plant is shown in Fig. 3.

4.2. Linear system

Let us now consider the second order continuous-time linear plant of the type defined in Eq. (35) as

$$\dot{x}(t) = A(t)x(t) + Bu(t) \tag{50}$$

where the time-varying plant parameters have the following form

$$\begin{aligned} a_{11} &= 1(1 + 0.1\cos(1.5t)) \quad a_{21} = 0.02(1 + 0.05\cos(0.75t)) \\ a_{12} &= 0.04(1 + 0.05\cos(0.75t)) \quad a_{22} = 2(1 + 0.1\cos(1.5t)) \end{aligned} \tag{51}$$

and the matrix B is defined by $b_{11} = 2$, $b_{21} = 0$, $b_{12} = 0.02$ and $b_{22} = 1.5$. The plant initial conditions were set as $x(0) = [0.1 \quad 0.4]^T \in \Re^2$.

In this case the controller is defined by Eqs. (39)–(41) together with the adaptive laws (42) and (43). The controller parameters were chosen as $\varepsilon = 0.01$ and $\gamma = 100$, and the initial conditions for the parameter estimates are $\beta(0) = 0.1$ and $K(0) = \begin{bmatrix} 0.5 & 0.02 \\ 0.002 & -0.18 \end{bmatrix}$.

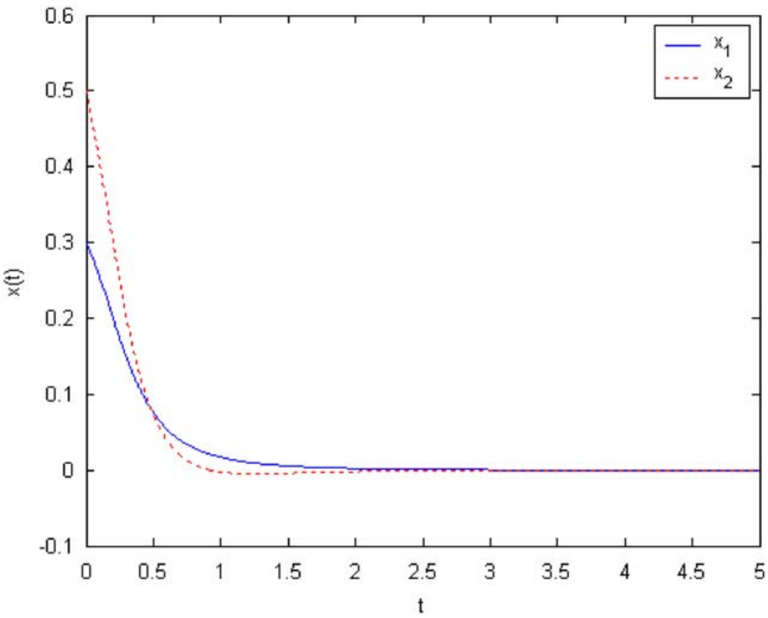


Fig. 1. Simulation of the nonlinear time-varying system. Evolution of the plant state.

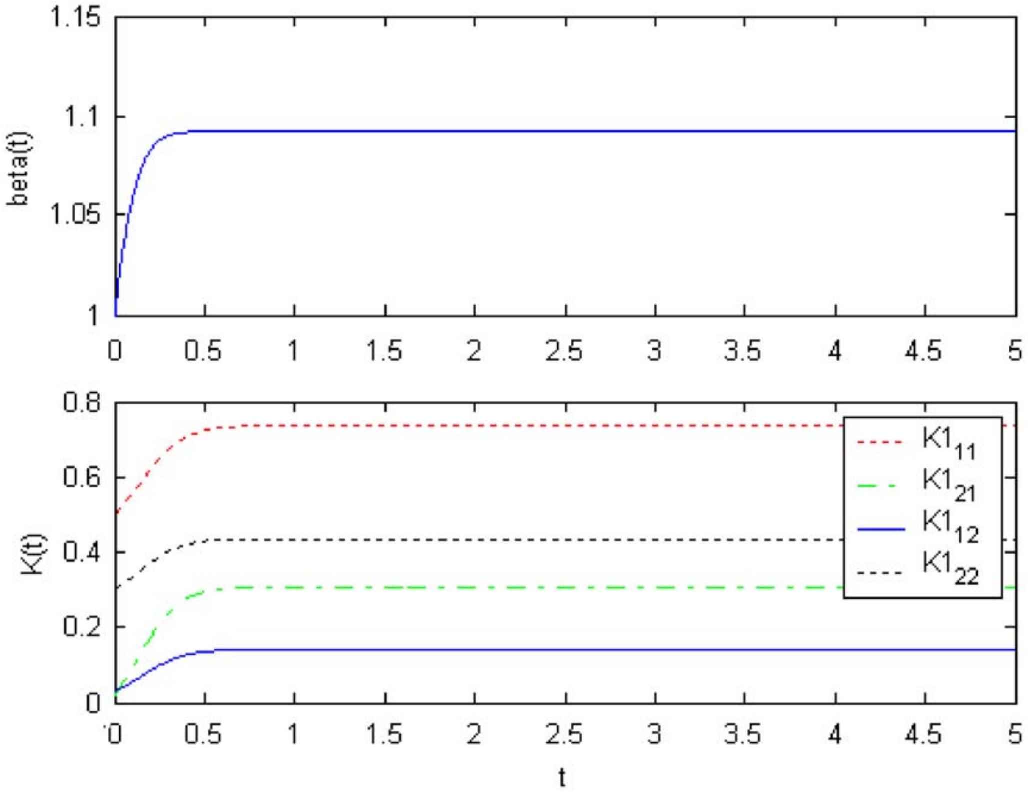


Fig. 2. Simulation of the nonlinear time-varying system. Evolution of the control parameters ($\hat{\beta}$, K_1).

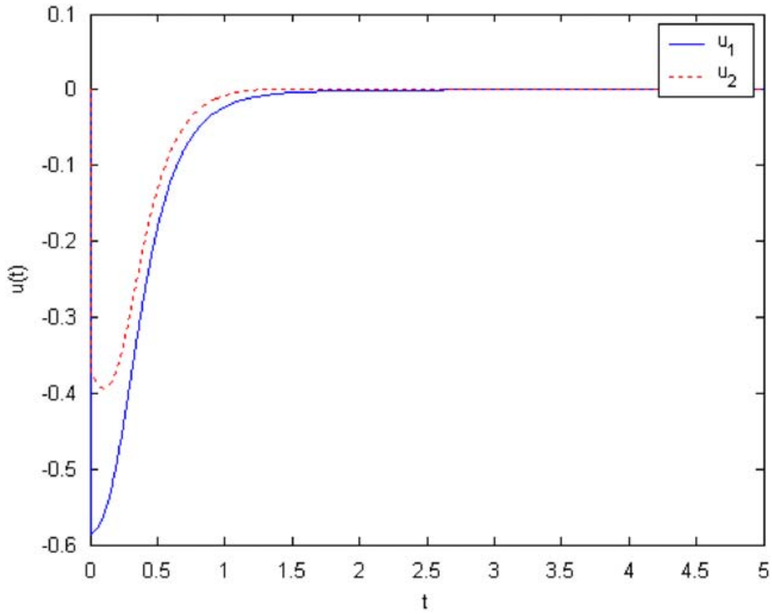


Fig. 3. Simulation of the nonlinear time-varying system. Evolution of the control input.

In Fig. 4, the evolution of the state of the plant is plotted. Like in the nonlinear case, it can be observed that the state goes to zero as t goes to infinity. The behavior of the controller parameters $\beta(t)$ and $K(t)$ are shown in Fig. 5. As expected, the parameter $\beta(t)$ evolves from its initial value up to a final bounded value determined by the stability conditions of the adaptive system. Finally, the evolution of the control input applied to the plant is shown in Fig. 6.

In all the simulations presented in this section the behavior observed is as expected and as predicted by the theoretical results exposed in Sections 2 and 3.

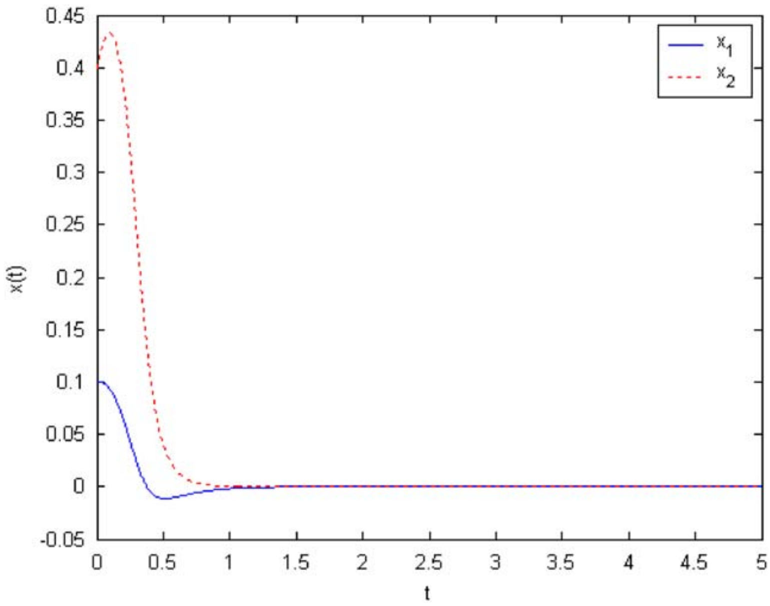


Fig. 4. Simulation of the linear time-varying system. Evolution of the plant state.

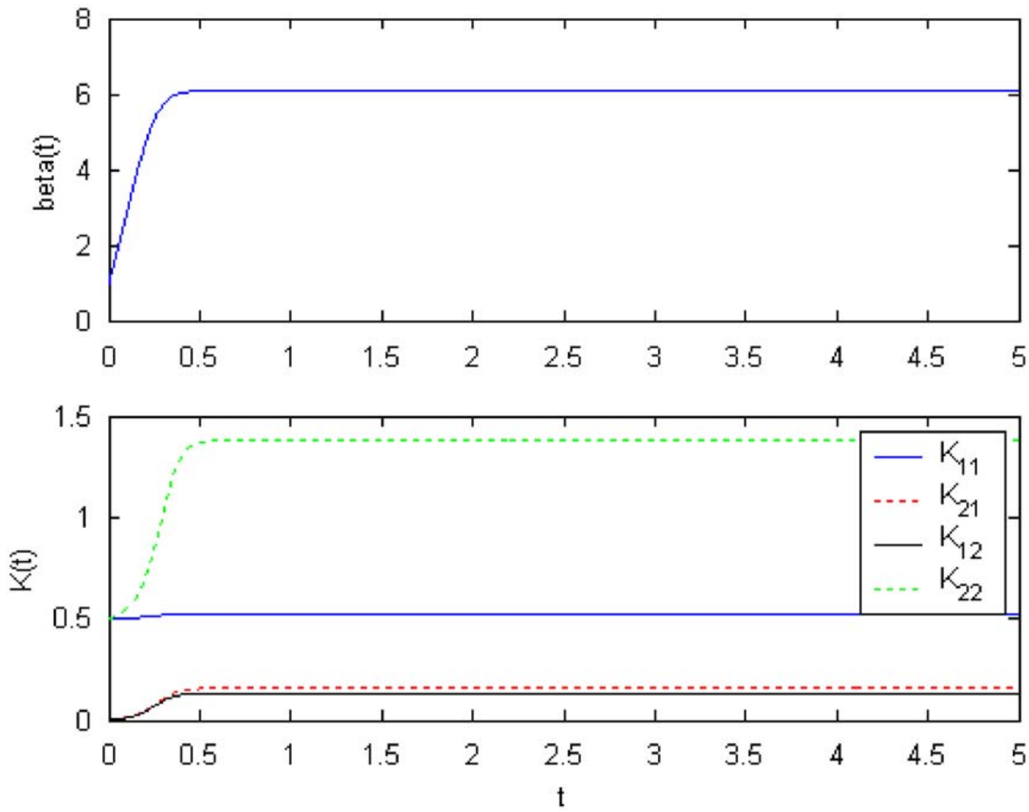


Fig. 5. Simulation of the linear time-varying system. Evolution of the control parameters ($\hat{\beta}$, K).

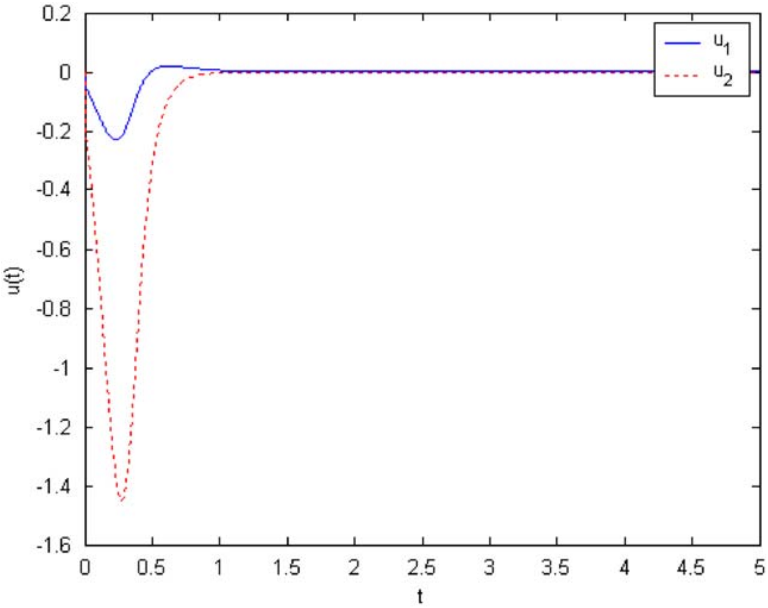


Fig. 6. Simulation of the linear time-varying system. Evolution of the control input.

5. Conclusions

Using Lyapunov's stability theory a new scheme for adaptive stabilization of time-varying linear and nonlinear plants was designed. This control scheme allows that the state of the plant with bounded time-varying parameters converge asymptotically to zero.

For the nonlinear case given by Eq. (1) having n^2+n unknown parameters (n time-varying and n^2 constant unknown parameters), when the matrix B is unknown the controller has to adjust n^2+1 parameters and the proposed scheme provides only local stability results. On the contrary, when the matrix B is known only one parameter has to be adjusted and the proposed scheme provides global stability results.

For the linear case given by Eq. (35) having $2n^2$ unknown parameters (n^2 time-varying and n^2 constant unknown parameters), when the matrix B is unknown the controller has to adjust n^2+1 parameters and the proposed scheme provides only local stability results. On the contrary, when the matrix B is known only one parameter has to be adjusted and the proposed scheme provides global stability results.

In order to verify the behavior of the controller based on Theorems 1 and 2, a set of simulations were presented for both, linear and nonlinear second-order systems assuming that matrix B is unknown, and was found that the simulation results are in complete agreement with the theoretically expected results presented in Sections 2 and 3.

Simulation results (not shown here for the sake of space) indicate that the overall adaptive system performs quite well and the use of adaptive gains in the adaptive laws can be introduced to modify the transient behavior and the convergence to zero.

Acknowledgement

The results reported in this paper have been supported by CONICYT-CHILE through grants FONDECYT no. 1030962 and FONDECYT no. 1061170.

References

[1] K.S. Narendra, Y.H. Lin, L.S. Valavani, Stable adaptive controller design, Part II: proof of stability, IEEE Transactions on Automatic Control 25 (1980) 440–449.

[2] G.C. Goodwin, P.J. Ramadge, P.E. Caines, Discrete time multi-variable adaptive control, IEEE Transactions on Automatic Control 25 (1980) 449–456.

[3] X. Xie, R.J. Evans, Discrete time adaptive control for deterministic time-varying systems, Automatica 20 (3) (1984) 309–319.

[4] F. Ohkawa, Model reference adaptive control system for discrete linear time-varying systems with periodically varying parameters and time-delay, International Journal of Control 44 (1) (1986) 171–179.

[5] R.H. Middleton, G.C. Goodwin, Adaptive control of time-varying linear systems, IEEE Transactions on Automatic Control 33 (2) (1988) 150–155.

[6] C. Tsakalis, P.A. Ioannou, A new indirect adaptive control scheme for time-varying plants, IEEE Transactions on Automatic Control 35 (6) (1990) 667–705.

[7] R. Marino, P. Tomei, An adaptive output feedback control for a class of nonlinear systems with time-varying parameters, IEEE Transactions on Automatic Control 44 (11) (1999) 2190–2194.

[8] S.S. Ge, J. Wang, Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients, IEEE Transactions on Automatic Control 48 (8) (2003) 1463–1469.

[9] C. Wang, S.S. Ge, Adaptive synchronization of uncertain chaotic systems via backstepping design, Chaos, Solitons and Fractals 12 (2001) 1199–1206.

[10] T.L. Liao, S.H. Tsai, Adaptive synchronization of chaotic systems and its application to secure communications, Chaos, Solitons and Fractals 11 (2000) 1387–1396.

[11] L. Zhi, C.Z. Han, Adaptive control and identification of chaotic systems, Chinese Physics 10 (6) (2001) 494–496.

[12] L. Zhi, C.Z. Han, Global adaptive synchronization of chaotic systems with uncertain parameters, Chinese Physics 11 (1) (2002) 9–11.

[13] Z. Li, C.H. Han, S. Shi, Modification for synchronization of Rossler and Chen chaotic systems, Physics Letters A 301 (2002) 224–230.

[14] C. Wang, S.S. Ge, Synchronization of two uncertain chaotic systems via adaptive backstepping, International Journal of Bifurcation and Chaos 11 (6) (2001) 1743–1751.

[15] C. Wang, S.S. Ge, Adaptive backstepping control of uncertain Lorenz system, International Journal of Bifurcation and Chaos 11 (4) (2001) 1115–1119.

[16] Z. Li, G. Chen, S. Shi, C.H. Han, Robust adaptive tracking control for a class of uncertain chaotic systems, Physics Letters A 310 (2003) 40–43.

[17] J. Estrada, M.A. Duarte-Mermoud, Simplification of a control methodology for a class of uncertain chaotic systems, WSEAS Transactions on Electronics 1 (2004) 347–352.

[18] S. Ge, C. Wang, Adaptive backstepping control of a class of chaotic systems, in: Proceedings of the 38 IEEE Conference on Decision and Control, December 1999, pp. 714–719.

[19] Y.P. Tian, X. Yu, Adaptive control of chaotic dynamical systems using invariant manifold approach, IEEE Transactions on Circuits and Systems I 47 (10) (2000) 1537–1542.

[20] J.L. Estrada, M.A. Duarte-Mermoud, J.C. Travieso-Torres, N.H. Beltrán, Simplified robust adaptive control of a class of time-varying chaotic systems, COMPEL: The International Journal for Computation & Mathematics in Electrical & Electronic Engineering 27 (2) (2008) 511–519.

[21] J.L. Estrada, M.A. Duarte-Mermoud, J.C. Travieso-Torres, A new model reference adaptive control for nonlinear time varying plants, in: R. Castro-Linares (Ed.), Proceedings of the 7th IASTED International Conference on Control & Applications (CA 2005), May 18–20, 2005, Cancún, México. Proceedings in CD, Track No. 460-063. 6pp.

[22] E. Ott, C. Grebogi, J.A. Yorke, Controlling chaos, Physical Review Letters 64 (1990) 1196–1199.

[23] M.A. Duarte-Mermoud, R. Castro-Linares, A. Castillo-Facuse, Direct passivity of a class of MIMO nonlinear systems using adaptive feedback, International Journal of Control 75 (1) (2002) 23–33.

[24] K.S. Narendra, A.M. Annawamy, Stable Adaptive Control, Prentice-Hall, Englewood, NJ, 1989.

[25] M.A. Duarte-Mermoud, R. Castro-Linares, A. Castillo-Facuse, Adaptive passivity of nonlinear systems using time-varying gains, Dynamics and Control 11 (4) (2001) 333–351.

[26] M.A. Duarte-Mermoud, J.M. Méndez-Miquel, R. Castro-Linares, A. Castillo-Facuse, Adaptive passivation with time-varying gains of MIMO nonlinear systems, Kybernetes 32 (9/10) (2003) 1342–1368.